CSC D70:
Compiler Optimization
Prefetching

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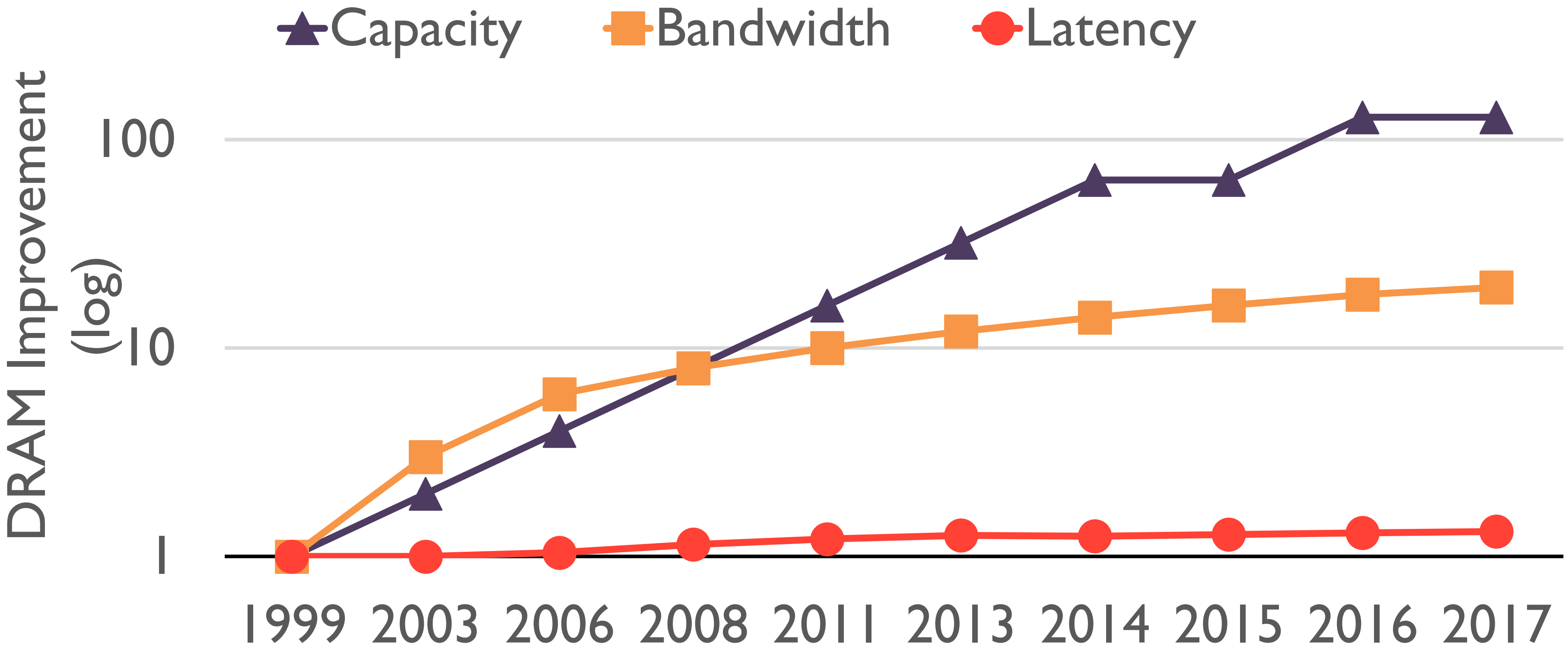
Winter 2018

The content of this lecture is adapted from the lectures of Todd Mowry and Phillip Gibbons

The Memory Latency Problem



- ↑ processor speed >> ↑ memory speed
- caches are not a panacea

Prefetching for Arrays: Overview

- Tolerating Memory Latency
- Prefetching Compiler Algorithm and Results
- Implications of These Results

Coping with Memory Latency

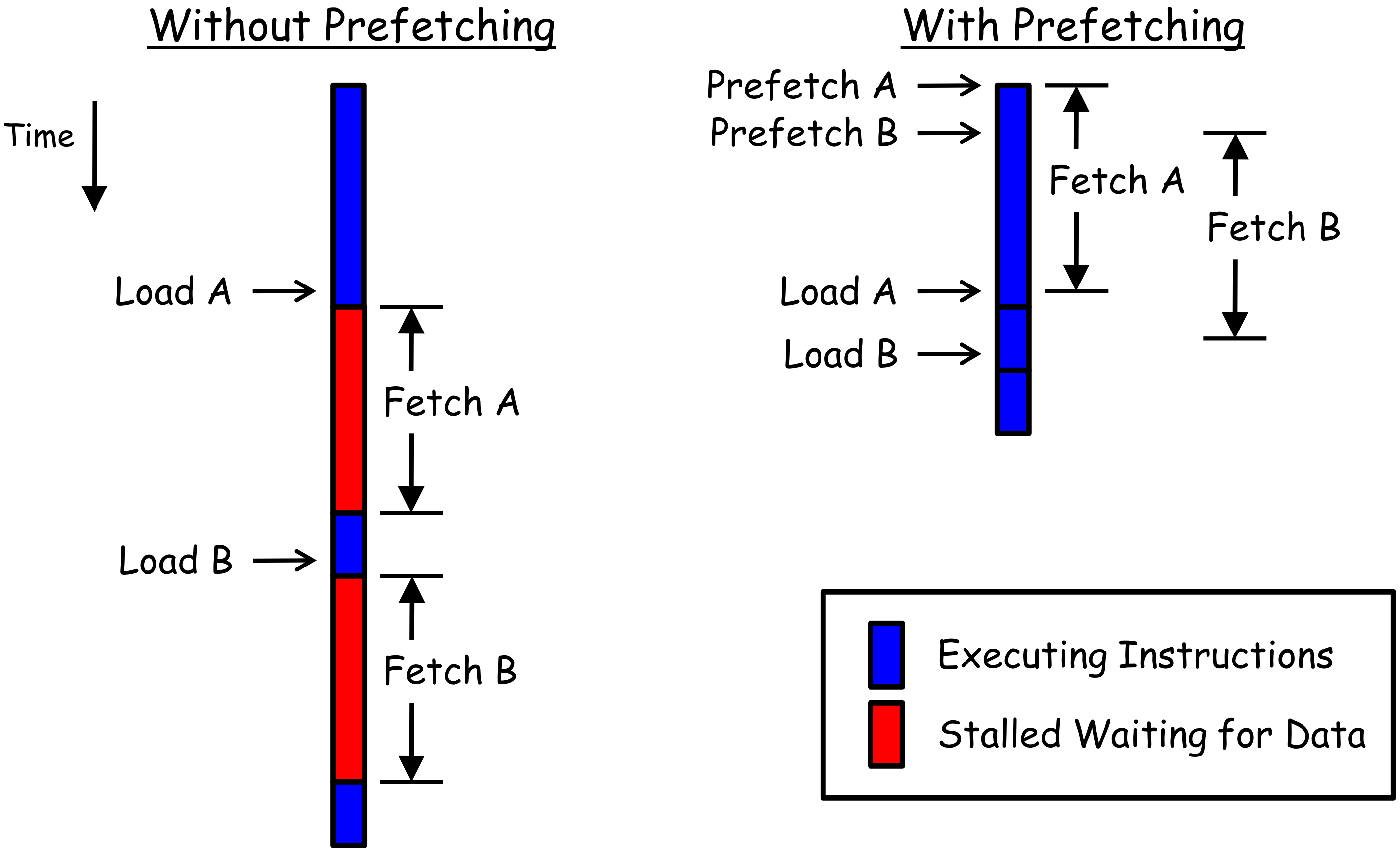
Reduce Latency:

- Locality Optimizations
  - reorder iterations to improve cache reuse

Tolerate Latency:

- Prefetching
  - move data close to the processor before it is needed

Tolerating Latency Through Prefetching



- overlap memory accesses with computation and other accesses

Types of Prefetching

Cache Blocks:

- (-) limited to unit-stride accesses

Nonblocking Loads:

- (-) limited ability to move back before use

Hardware-Controlled Prefetching:

- (-) limited to constant-strides and by branch prediction
- (+) no instruction overhead

Software-Controlled Prefetching:

- (-) software sophistication and overhead
- (+) minimal hardware support and broader coverage

Prefetching Goals

- Domain of Applicability
- Performance Improvement
  - maximize benefit
  - minimize overhead

Prefetching Concepts

possible only if addresses can be determined ahead of time
coverage factor = fraction of misses that are prefetched
unnecessary if data is already in the cache
effective if data is in the cache when later referenced

Analysis: what to prefetch

- maximize coverage factor
- minimize unnecessary prefetches

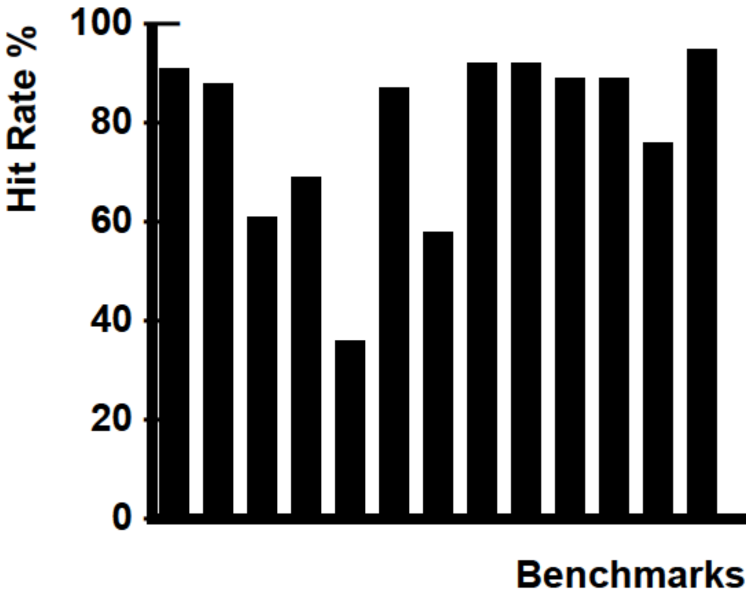
Scheduling: when/how to schedule prefetches

- maximize effectiveness
- minimize overhead per prefetch

Reducing Prefetching Overhead

- instructions to issue prefetches
- extra demands on memory system

Hit Rates for Array Accesses



- important to minimize unnecessary prefetches

Compiler Algorithm

Analysis: what to prefetch

- Locality Analysis

Scheduling: when/how to issue prefetches

- Loop Splitting
- Software Pipelining

Steps in Locality Analysis

1. Find data reuse
    - if caches were infinitely large, we would be finished
2. Determine "localized iteration space"
    - set of inner loops where the data accessed by an iteration is expected to fit within the cache
3. Find data locality:
    - reuse $\cap$ localized iteration space $\Rightarrow$ locality

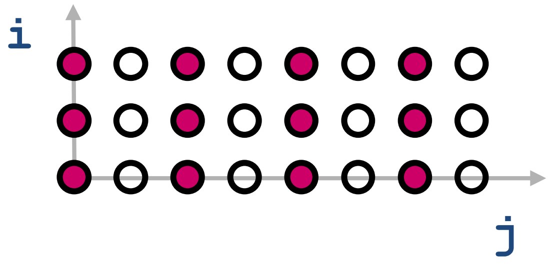
Data Locality Example

```
for i = 0 to 2
  for j = 0 to 100
    A[i][j] = B[j][0] + B[j+1][0];
```

○ Hit

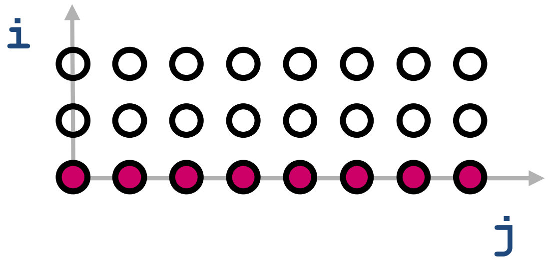
● Miss

A[i][j]



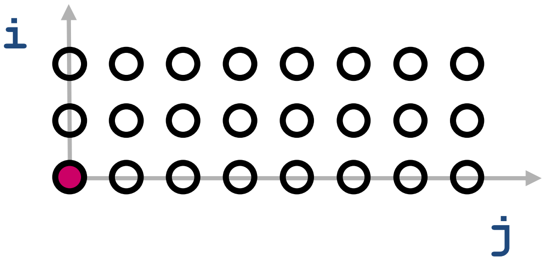
Spatial

B[j+1][0]



Temporal

B[j][0]



Group

Reuse Analysis: Representation

```
for i = 0 to 2
 for j = 0 to 100
  A[i][j] = B[j][0] + B[j+1][0];
```

- Map *n* loop indices into *d* array indices via array indexing function:

$$\vec{f}(\vec{\imath}) = H\vec{\imath} + \vec{c}$$

$$\begin{aligned}
\texttt{A[i][j]} &= \texttt{A}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) \\
\texttt{B[j][0]} &= \texttt{B}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) \\
\texttt{B[j+1][0]} &= \texttt{B}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)
\end{aligned}$$

Finding Temporal Reuse

- Temporal reuse occurs between iterations $\vec{\imath}_1$ and $\vec{\imath}_2$ whenever:

$$H\vec{\imath}_1 + \vec{c} = H\vec{\imath}_2 + \vec{c}$$

$$H(\vec{\imath}_1 - \vec{\imath}_2) = \vec{0}$$

- Rather than worrying about individual values $\vec{\imath}_1$ of $\vec{\imath}_2$ and, we say that reuse occurs along direction $\vec{r}$ vector when:

$$H(\vec{r}) = \vec{0}$$

- Solution: compute the *nullspace* of *H*

Temporal Reuse Example

```
for i = 0 to 2
 for j = 0 to 100
  A[i][j] = B[j][0] + B[j+1][0];
```

- Reuse between iterations (i_1,j_1) and (i_2,j_2) whenever:

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ j_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_2 \\ j_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 - i_2 \\ j_1 - j_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- True whenever $j_1 = j_2$, and regardless of the difference between i_1 and i_2.
  - i.e. whenever the difference lies along the nullspace of $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$
  - which is span{(1,0)} (i.e. the outer loop).

Prefetch Predicate

Locality Type	Miss Instance	Predicate
None	Every Iteration	True
Temporal	First Iteration	$i = 0$
Spatial	Every l iterations (l = cache line size)	$(i \bmod l) = 0$

Example:

```
for i = 0 to 2
  for j = 0 to 100
    A[i][j] = B[j][0] + B[j+1][0];
```

Reference	Locality	Predicate
`A[i][j]`	$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} \text{none} \\ \text{spatial} \end{bmatrix}$	$(j \bmod 2) = 0$
`B[j+1][0]`	$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} \text{temporal} \\ \text{none} \end{bmatrix}$	$i = 0$

Compiler Algorithm

Analysis: what to prefetch

- Locality Analysis

Scheduling: when/how to issue prefetches

- Loop Splitting
- Software Pipelining

Loop Splitting

- Decompose loops to isolate cache miss instances
  - cheaper than inserting IF statements

Locality Type	**Predicate**	**Loop Transformation**
None	True	None
Temporal	`i` = 0	Peel loop `i`
Spatial	(`i` mod l) = 0	Unroll loop `i` by l

- Apply transformations recursively for nested loops
- Suppress transformations when loops become too large
  - avoid code explosion

Software Pipelining

$$\text{Iterations Ahead} = \left\lceil \frac{l}{s} \right\rceil$$

where l = memory latency, s = shortest path through loop body

Original Loop

```
for (i = 0; i<100; i++)
   a[i] = 0;
```

Software Pipelined Loop
(5 iterations ahead)

```
for (i = 0; i<5; i++)        /* Prolog */
   prefetch(&a[i]);

for (i = 0; i<95; i++) { /* Steady State*/
   prefetch(&a[i+5]);
   a[i] = 0;
}

for (i = 95; i<100; i++) /* Epilog */
   a[i] = 0;
```

Example Revisited

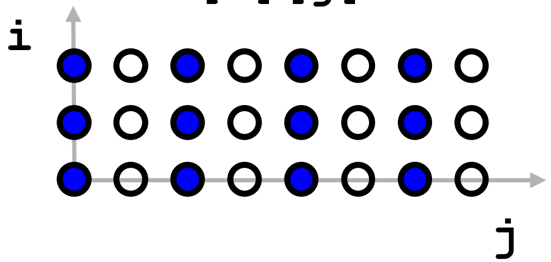
Original Code

```
for (i = 0; i < 3; i++)
  for (j = 0; j < 100; j++)
    A[i][j] = B[j][0] + B[j+1][0];
```

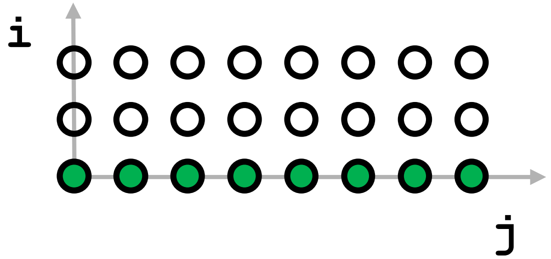
○ Cache Hit

● ● Cache Miss

A[i][j]



B[j+1][0]



Code with Prefetching

i = 0

```
prefetch(&A[0][0]);
for (j = 0; j < 6; j += 2) {
  prefetch(&B[j+1][0]);
  prefetch(&B[j+2][0]);
  prefetch(&A[0][j+1]);
}
for (j = 0; j < 94; j += 2) {
  prefetch(&B[j+7][0]);
  prefetch(&B[j+8][0]);
  prefetch(&A[0][j+7]);
  A[0][j]  = B[j][0]+B[j+1][0];
  A[0][j+1] = B[j+1][0]+B[j+2][0];
}
for (j = 94; j < 100; j += 2) {
  A[0][j]  = B[j][0]+B[j+1][0];
  A[0][j+1] = B[j+1][0]+B[j+2][0];
}
```

i > 0

```
for (i = 1; i < 3; i++) {
  prefetch(&A[i][0]);
  for (j = 0; j < 6; j += 2)
    prefetch(&A[i][j+1]);
  for (j = 0; j < 94; j += 2) {
    prefetch(&A[i][j+7]);
    A[i][j]  = B[j][0] + B[j+1][0];
    A[i][j+1] = B[j+1][0] + B[j+2][0];
  }
  for (j = 94; j < 100; j += 2) {
    A[i][j]  = B[j][0] + B[j+1][0];
    A[i][j+1] = B[j+1][0] + B[j+2][0];
  }
}
```

Prefetching Indirections

```
for (i = 0; i<100; i++)
   sum += A[index[i]];
```

Analysis: what to prefetch

- both dense and indirect references
- difficult to predict whether indirections hit or miss

Scheduling: when/how to issue prefetches

- modification of software pipelining algorithm

Software Pipelining for Indirections

Original Loop

```
for (i = 0; i<100; i++)
   sum += A[index[i]];
```

Software Pipelined Loop
(5 iterations ahead)

```
for (i = 0; i<5; i++)      /* Prolog 1 */
   prefetch(&index[i]);

for (i = 0; i<5; i++) {    /* Prolog 2 */
   prefetch(&index[i+5]);
   prefetch(&A[index[i]]);
}
for (i = 0; i<90; i++) { /* Steady State*/
   prefetch(&index[i+10]);
   prefetch(&A[index[i+5]]);
   sum += A[index[i]];
}
for (i = 90; i<95; i++) { /* Epilog 1 */
   prefetch(&A[index[i+5]]);
   sum += A[index[i]];
}
for (i = 95; i<100; i++)   /* Epilog 2 */
   sum += A[index[i]];
```

Summary of Results

Dense Matrix Code:

- eliminated 50% to 90% of memory stall time
- overheads remain low due to prefetching selectively
- significant improvements in overall performance (6 over 45%)

Indirections, Sparse Matrix Code:

- expanded coverage to handle some important cases

Prefetching for Arrays: Concluding Remarks

- Demonstrated that software prefetching is effective
  - selective prefetching to eliminate overhead
  - dense matrices and indirections / sparse matrices
  - uniprocessors and multiprocessors

- Hardware should focus on providing sufficient memory bandwidth

Prefetching for Recursive Data Structures

Recursive Data Structures

- Examples:
  - linked lists, trees, graphs, ...
- A common method of building large data structures
  - especially in non-numeric programs
- Cache miss behavior is a concern because:
  - large data set with respect to the cache size
  - temporal locality may be poor
  - little spatial locality among consecutively-accessed nodes

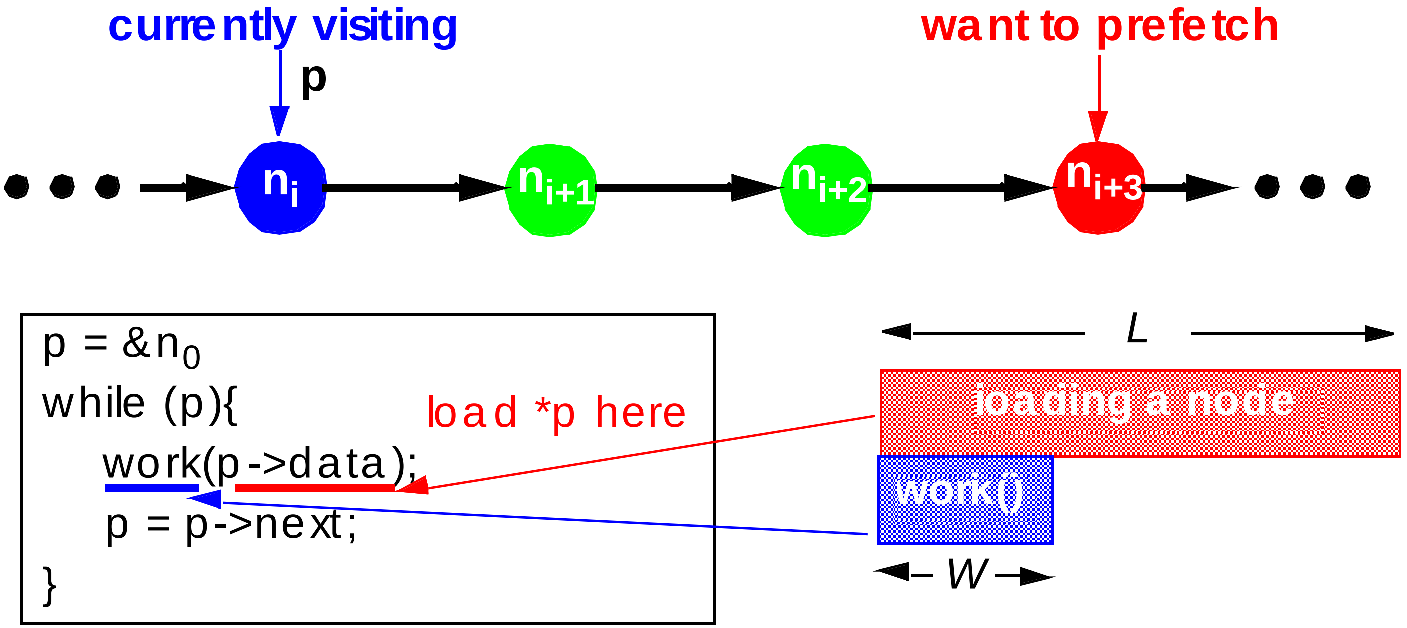
Goal:

- Automatic Compiler-Based Prefetching for Recursive Data Structures

Overview

- Challenges in Prefetching Recursive Data Structures
- Three Prefetching Algorithms
- Experimental Results
- Conclusions

Scheduling Prefetches for Recursive Data Structures



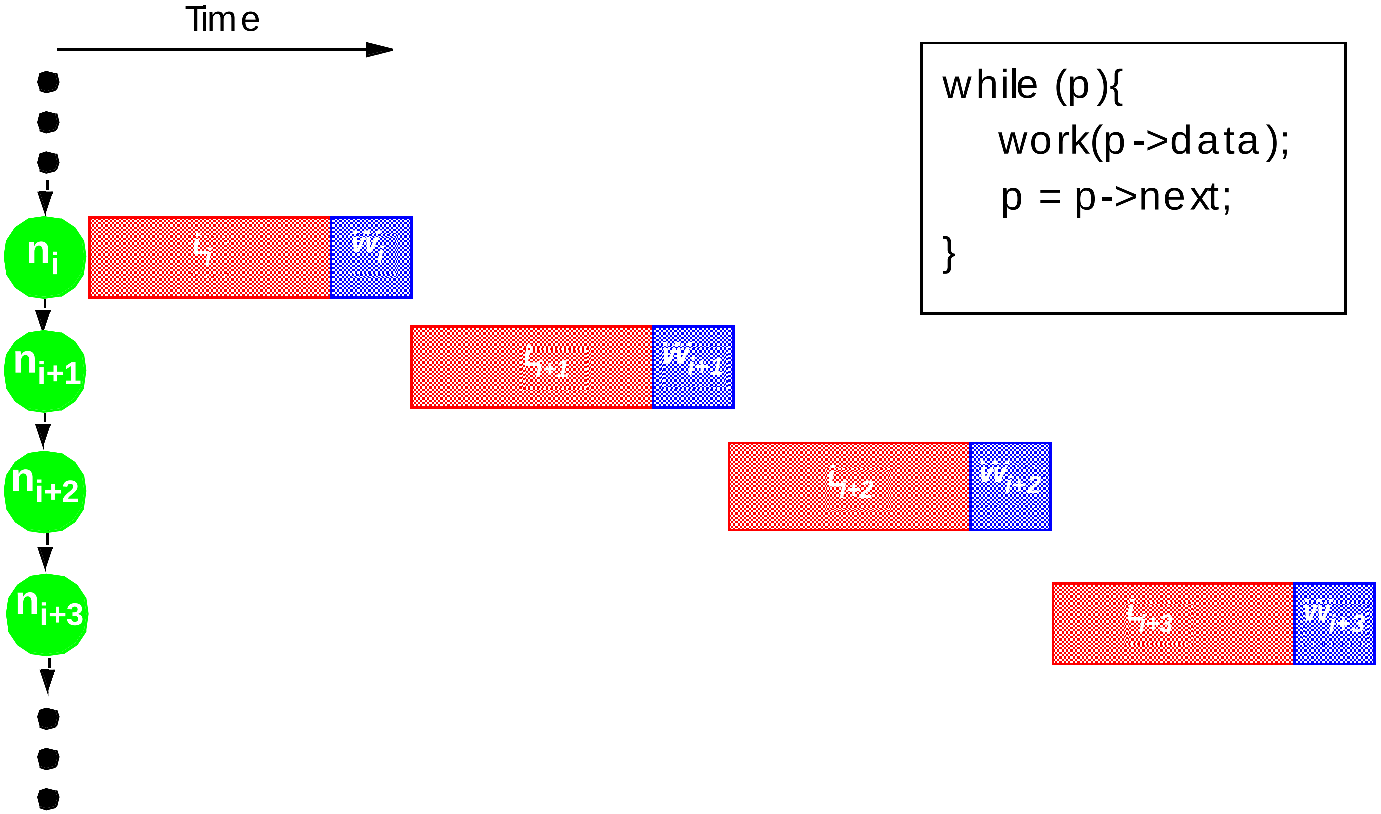
```
p = &n0
while (p){
    work(p->data);
    p = p->next;
}
```

Our Goal: *fully hide latency*

  - thus achieving fastest possible computation rate of 1/W

- e.g., if L = 3W, we must prefetch 3 nodes ahead to achieve this

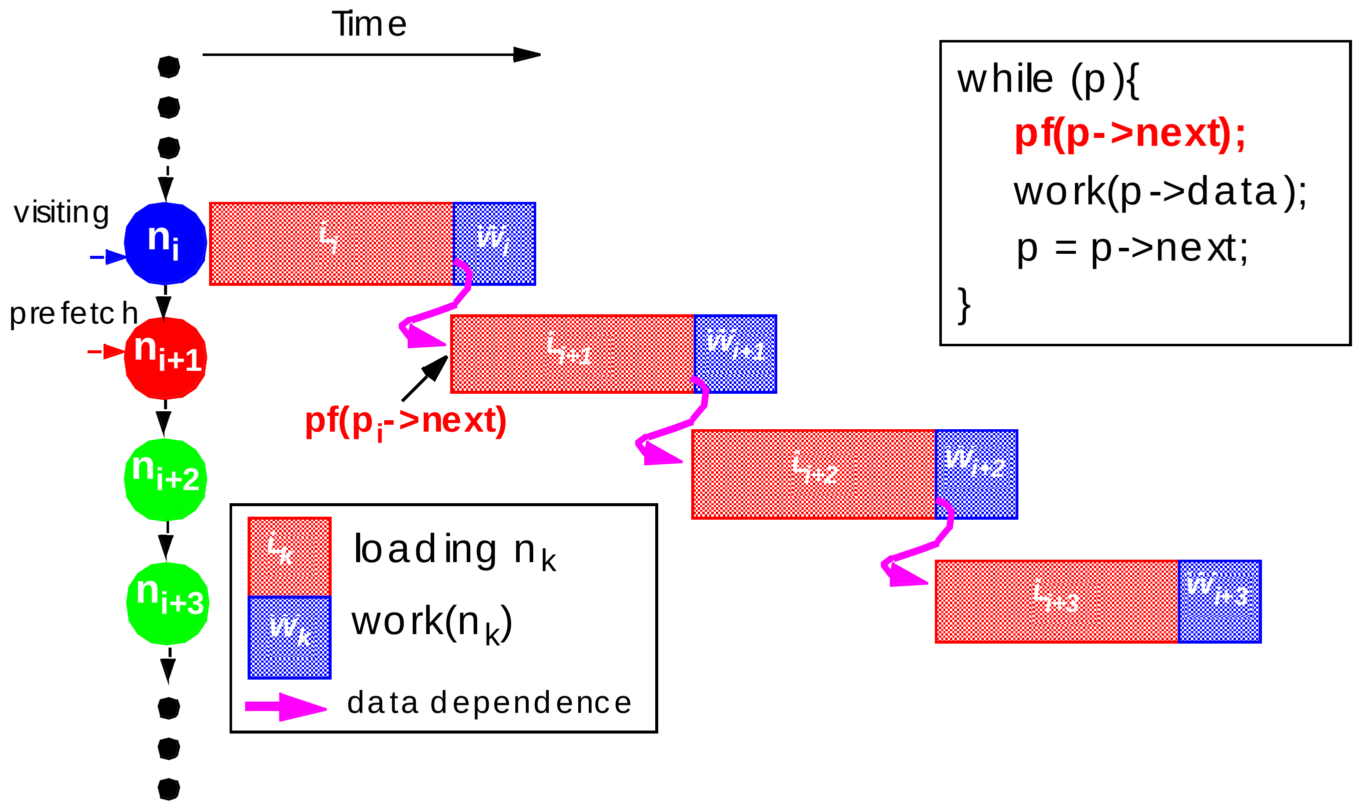
Performance without Prefetching



```
while (p){
    work(p->data);
    p = p->next;
}
```

computation rate = 1 / (L+W)

Prefetching One Node Ahead



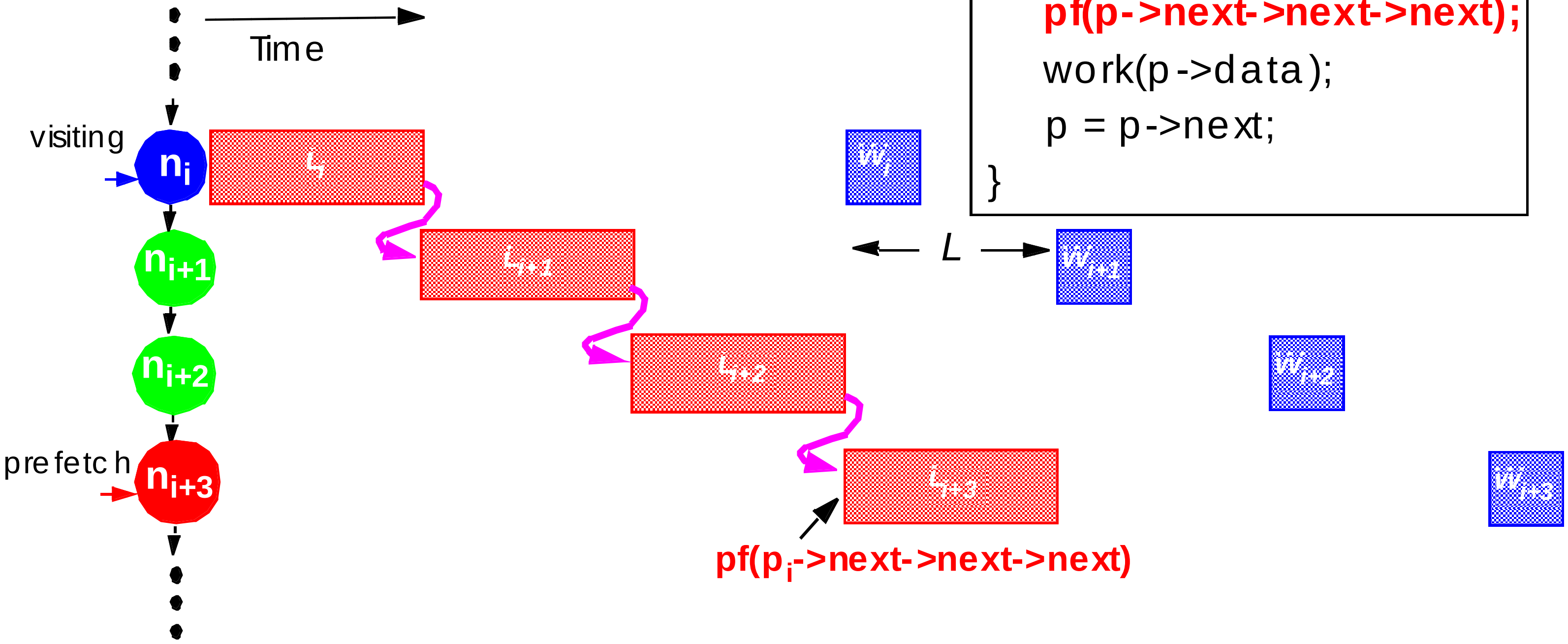
```
while (p){
    pf(p->next);
    work(p->data);
    p = p->next;
}
```

- Computation is overlapped with memory accesses

computation rate = 1/L

Prefetching Three Nodes Ahead

```
while (p){
    pf(p->next->next->next);
    work(p->data);
    p = p->next;
}
```

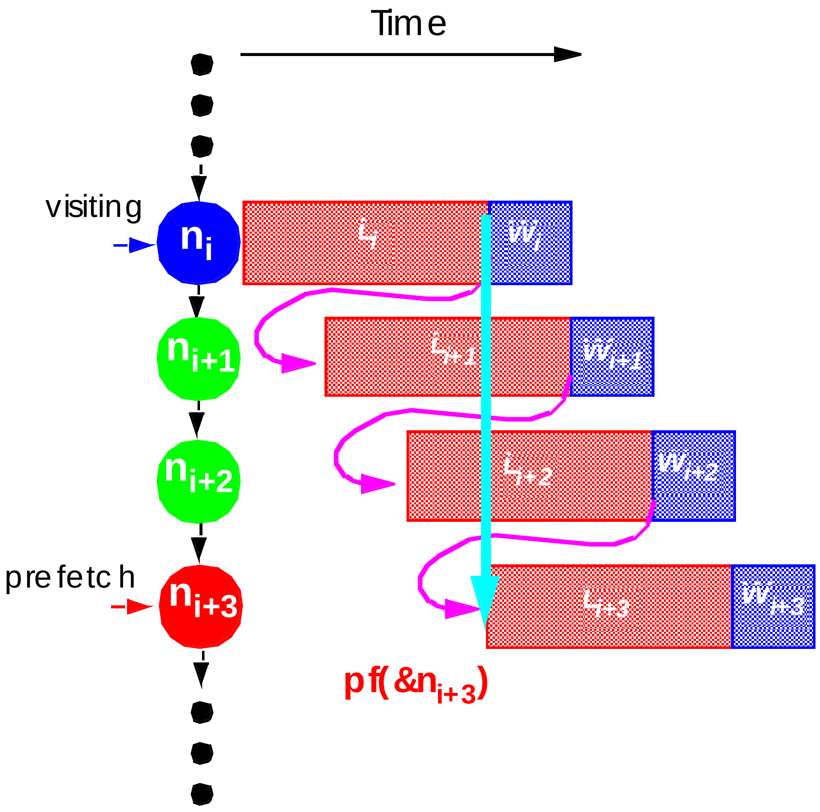


computation rate does not improve (still = 1/L)!

Pointer-Chasing Problem:

- any scheme which follows the pointer chain is limited to a rate of 1/L

Our Goal: Fully Hide Latency



```
while (p){
    pf(&n_{i+3});
    work(p->data);
    p = p->next;
}
```

- achieves the fastest possible computation rate of 1/W

Overview

- Challenges in Prefetching Recursive Data Structures
- Three Prefetching Algorithms
  - Greedy Prefetching
  - History-Pointer Prefetching
  - Data-Linearization Prefetching
- Experimental Results
- Conclusions

Pointer-Chasing Problem

Key:

- n_i needs to know $\&n_{i+d}$ without referencing the d-1 intermediate nodes

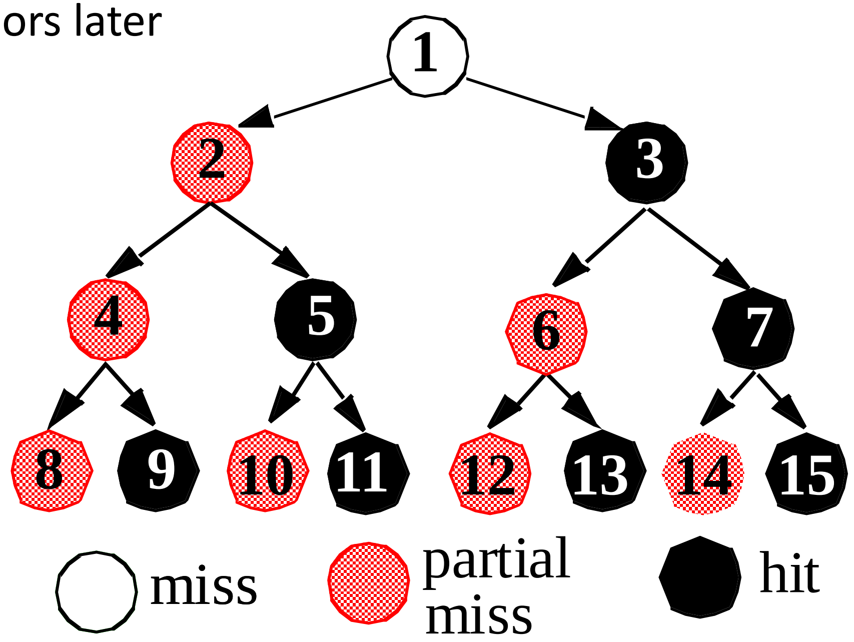
Our proposals:

- use *existing* pointer(s) in n_i to approximate $\&n_{i+d}$
  - Greedy Prefetching
- add *new* pointer(s) to n_i to approximate $\&n_{i+d}$
  - History-Pointer Prefetching
- compute $\&n_{i+d}$ *directly* from $\&n_i$ (no ptr deref)
  - History-Pointer Prefetching

Greedy Prefetching

- Prefetch all neighboring nodes (simplified definition)
  - only one will be followed by the immediate control flow
  - hopefully, we will visit other neighbors later

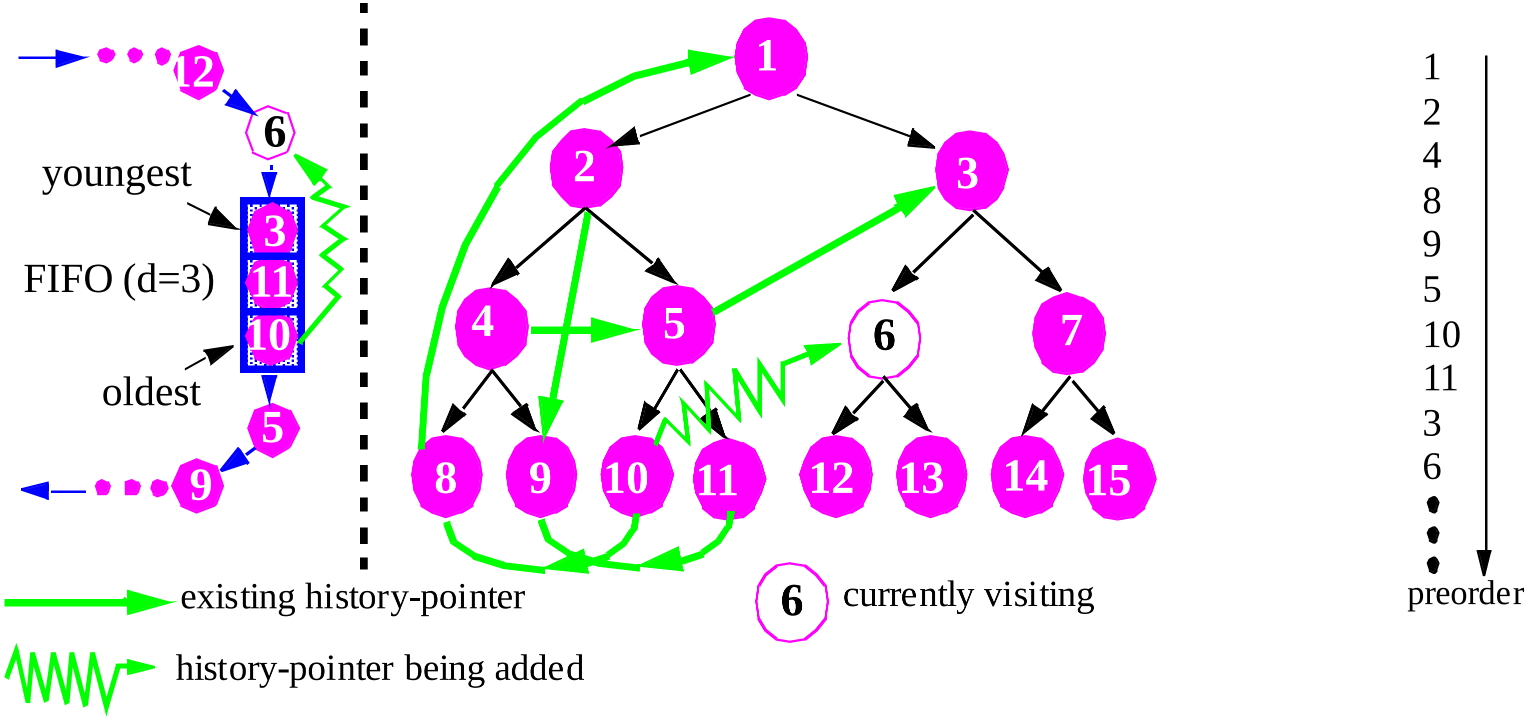
```
preorder(treeNode * t){
  if (t != NULL){
    pf(t->left);
    pf(t->right);
    process(t->data);
    preorder(t->left);
    preorder(t->right);
  }
}
```



- Reasonably effective in practice
- However, little control over the prefetching distance

History-Pointer Prefetching

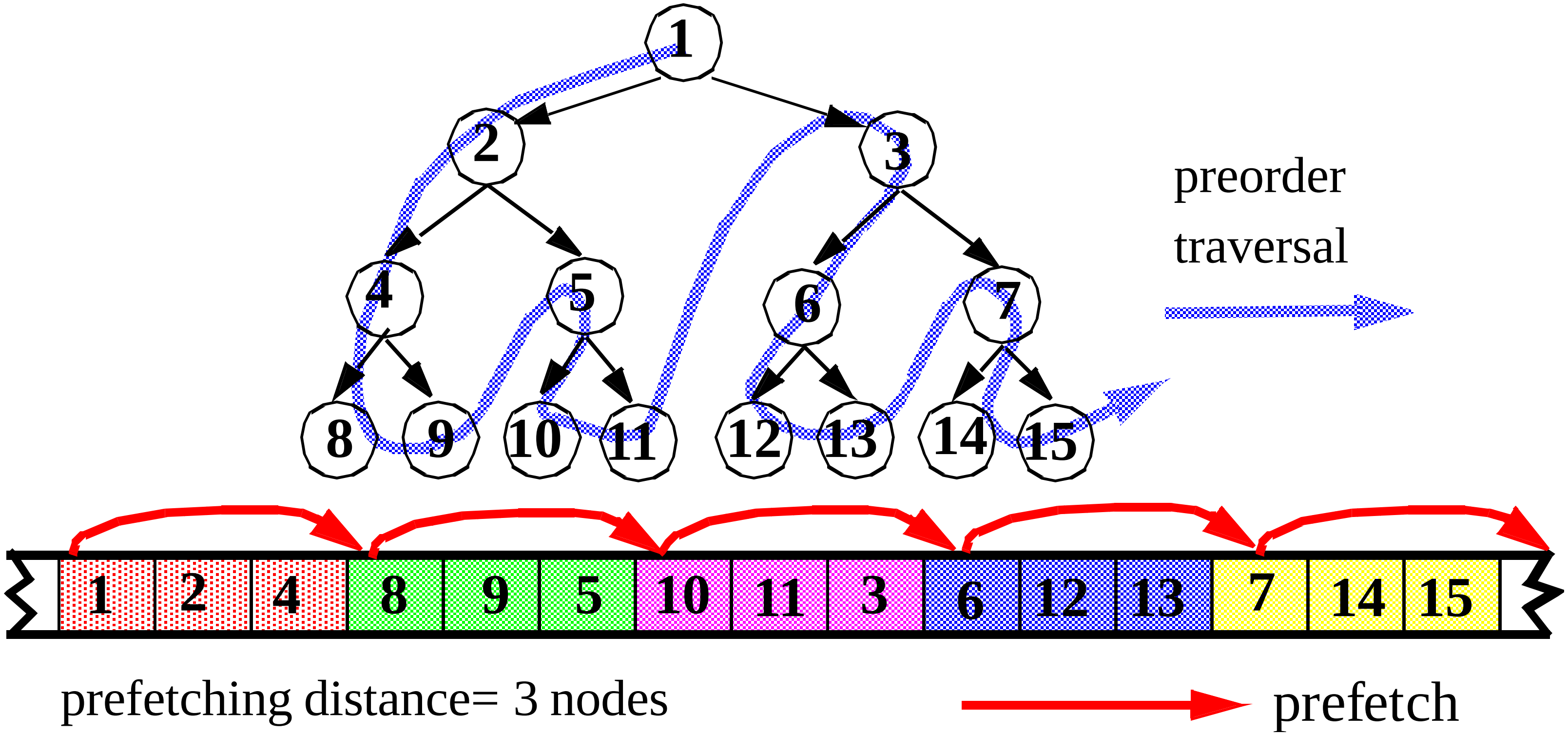
- Add new pointer(s) to each node
  - history-pointers are obtained from some recent traversal



- Trade space & time for better control over prefetching distances

Data-Linearization Prefetching

- No pointer dereferences are required
- Map nodes close in the traversal to contiguous memory



Summary of Prefetching Algorithms

	Greedy	History-Pointer	Data-Linearization
Control over Prefetching Distance	little	more precise	more precise
Applicability to Recursive Data Structures	any RDS	revisited; changes only slowly	must have a major traversal order; changes only slowly
Overhead in Preparing Prefetch Addresses	none	space + time	none in practice
Ease of Implementation	relatively straightforward	more difficult	more difficulty

Conclusions

- Propose 3 schemes to overcome the pointer-chasing problem:
  - Greedy Prefetching
  - History-Pointer Prefetching
  - Data-Linearization Prefetching
- Automated greedy prefetching in SUIF
  - improves performance significantly for half of Olden
  - memory feedback can further reduce prefetch overhead
- The other 2 schemes can outperform greedy in some situations

CSC D70:
Compiler Optimization
Parallelization

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Data Dependence

$$\begin{array}{ll} S_1: & A = 1.0 \\ S_2: & B = A + 2.0 \\ S_3: & A = C - D \\ & \vdots \\ S_4: & A = B/C \end{array}$$

We define four types of data dependence.

- Flow (true) dependence: a statement S_i precedes a statement S_j in execution and S_i computes a data value that S_j uses.
- Implies that S_i must execute before S_j.

$$S_i \, \delta^t \, S_j \qquad (S_1 \, \delta^t \, S_2 \quad \text{and} \quad S_2 \, \delta^t \, S_4)$$

Data Dependence

$$\begin{array}{ll} S_1 : & A = 1.0 \\ S_2 : & B = A + 2.0 \\ S_3 : & A = C - D \\ & \quad\vdots \\ S_4 : & A = B / C \end{array}$$

We define four types of data dependence.

- Anti dependence: a statement S_i precedes a statement S_j in execution and S_i uses a data value that S_j computes.
- It implies that S_i must be executed before S_j.

$$S_i \, \delta^a \, S_j \qquad (S_2 \, \delta^a \, S_3)$$

Data Dependence

$$\begin{array}{ll} S_1: & A = 1.0 \\ S_2: & B = A + 2.0 \\ S_3: & A = C - D \\ & \quad\vdots \\ S_4: & A = B/C \end{array}$$

We define four types of data dependence.

- Output dependence: a statement S_i precedes a statement S_j in execution and S_i computes a data value that S_j also computes.
- It implies that S_i must be executed before S_j.

$$S_i \, \delta^o \, S_j \qquad (S_1 \, \delta^o \, S_3 \quad \text{and} \quad S_3 \, \delta^o \, S_4)$$

Data Dependence

$$
\begin{array}{ll}
S_1: & A = 1.0 \\
S_2: & B = A + 2.0 \\
S_3: & A = C - D \\
 & \vdots \\
S_4: & A = B/C
\end{array}
$$

We define four types of data dependence.

- Input dependence: a statement S_i precedes a statement S_j in execution and S_i uses a data value that S_j also uses.
- Does this imply that S_i must execute before S_j?

$$S_i \, \delta^I \, S_j \qquad (S_3 \, \delta^I \, S_4)$$

Data Dependence (continued)

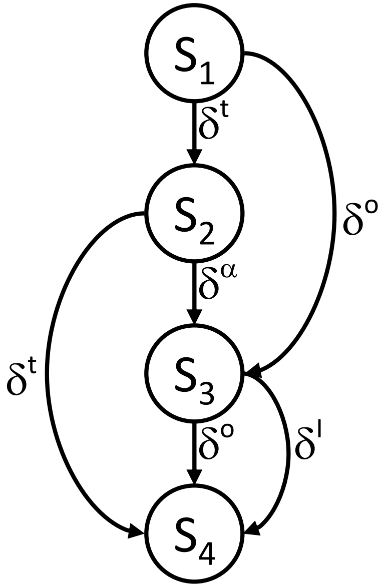
- The dependence is said to flow from S_i to S_j because S_i precedes S_j in execution.
- S_i is said to be the source of the dependence. S_j is said to be the sink of the dependence.
- The only “true” dependence is flow dependence; it represents the flow of data in the program.
- The other types of dependence are caused by programming style; they may be eliminated by re-naming.

$$\begin{array}{ll} S_1: & A = 1.0 \\ S_2: & B = A + 2.0 \\ S_3: & A1 = C - D \\ & \quad\vdots \\ S_4: & A2 = B/C \end{array}$$

Data Dependence (continued)

- Data dependence in a program may be represented using a dependence graph G=(V,E), where the nodes V represent statements in the program and the directed edges E represent dependence relations.

$$\begin{aligned} S_1 &: \quad A = 1.0 \\ S_2 &: \quad B = A + 2.0 \\ S_3 &: \quad A = C - D \\ &\qquad \vdots \\ S_4 &: \quad A = B/C \end{aligned}$$



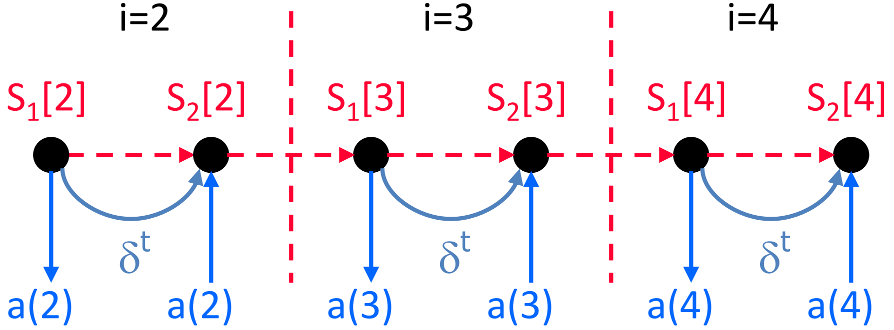
Value or Location?

- There are two ways a dependence is defined: value-oriented or location-oriented.

$$\begin{array}{ll} S_1: & A = 1.0 \\ S_2: & B = A + 2.0 \\ S_3: & A = C - D \\ & \quad\vdots \\ S_4: & A = B/C \end{array}$$

Example 1

```
      do i = 2, 4
S1:     a(i) = b(i) + c(i)
S2:     d(i) = a(i)
      end do
```

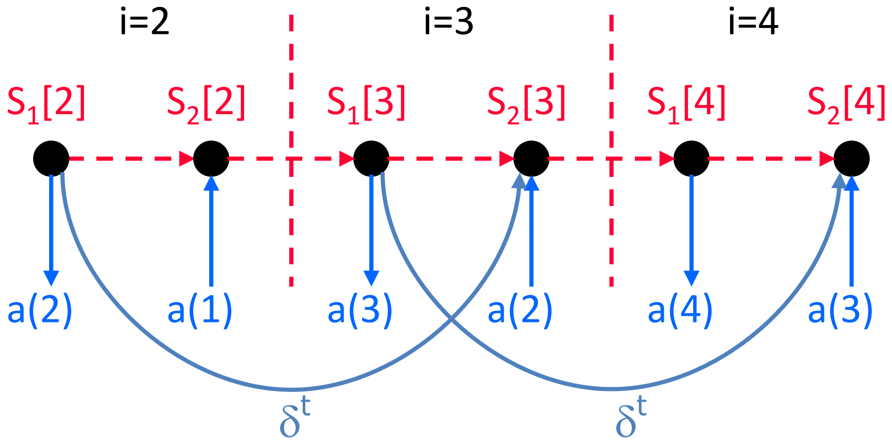


- There is an instance of S_1 that precedes an instance of S_2 in execution and S_1 produces data that S_2 consumes.
- S_1 is the source of the dependence; S_2 is the sink of the dependence.
- The dependence flows between instances of statements in the same iteration (loop-independent dependence).
- The number of iterations between source and sink (dependence distance) is 0. The dependence direction is =.

$$S_1 \delta^t_= S_2 \qquad \text{or} \qquad S_1 \delta^t_0 S_2$$

Example 2

```
    do i = 2, 4
S1:   a(i) = b(i) + c(i)
S2:   d(i) = a(i-1)
    end do
```

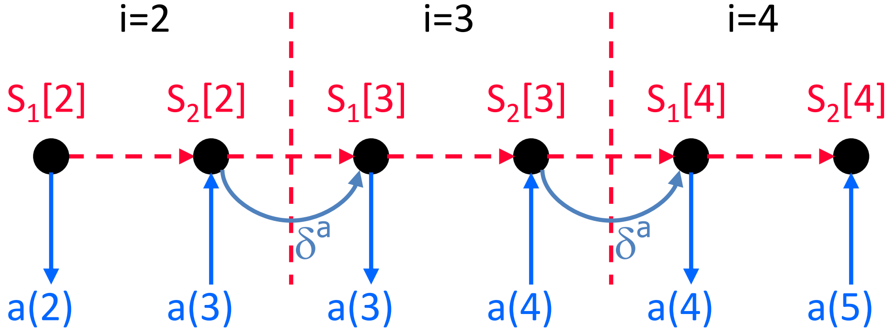


- There is an instance of S_1 that precedes an instance of S_2 in execution and S_1 produces data that S_2 consumes.
- S_1 is the source of the dependence; S_2 is the sink of the dependence.
- The dependence flows between instances of statements in different iterations (loop-carried dependence).
- The dependence distance is 1. The direction is positive (<).

$S_1 \, \delta^t_< \, S_2$ or $S_1 \, \delta^t_1 \, S_2$

Example 3

```
      do i = 2, 4
$S_1$:   a(i) = b(i) + c(i)
$S_2$:   d(i) = a(i+1)
      end do
```



- There is an instance of S_2 that precedes an instance of S_1 in execution and S_2 consumes data that S_1 produces.
- S_2 is the source of the dependence; S_1 is the sink of the dependence.
- The dependence is loop-carried.
- The dependence distance is 1.

$$S_2 \delta^a_< S_1 \quad \text{or} \quad S_2 \delta^a_1 S_1$$

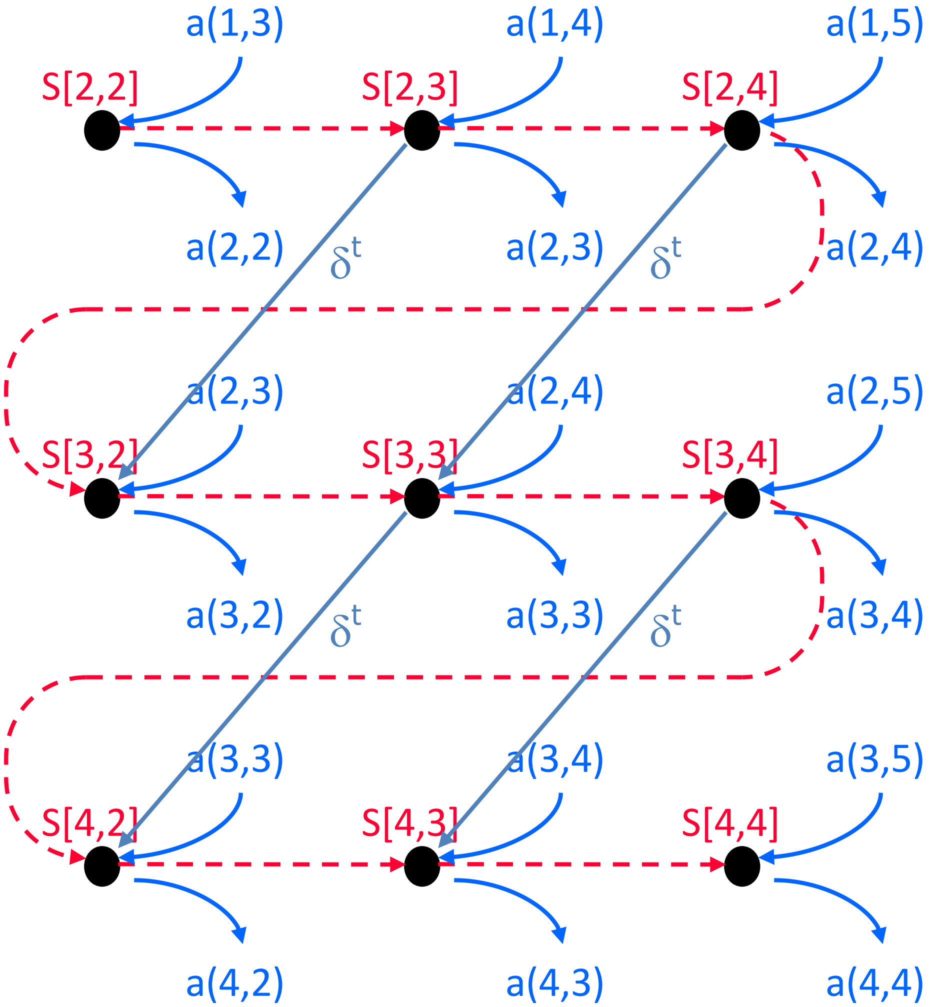
- Are you sure you know why it is $S_2 \delta^a_< S_1$ even though S_1 appears before S_2 in the code?

Example 4

```
    do i = 2, 4
      do j = 2, 4
S:      a(i,j) = a(i-1,j+1)
      end do
    end do
```

- An instance of S precedes another instance of S and S produces data that S consumes.
- S is both source and sink.
- The dependence is loop-carried.
- The dependence distance is (1,-1).

$S\,\delta^t_{(<,>)}\,S$ or $S\,\delta^t_{(1,-1)}\,S$



Problem Formulation

- Consider the following perfect nest of depth d:

```
do $I_1 = L_1, U_1$
   do $I_2 = L_2, U_2$
      ⋱
         do $I_d = L_d, U_d$
            $a(f_1(\vec{I}), f_2(\vec{I}), \cdots, f_m(\vec{I})) = \cdots$
            $\cdots = a(g_1(\vec{I}), g_2(\vec{I}), \cdots, g_m(\vec{I}))$
         enddo
      ⋰
   enddo
enddo
```

$$\overbrace{a(\quad, \underbrace{f_k(\vec{I})}_{\text{subscript function or subscript expression}}, \cdots, \quad)}^{\text{array reference}}$$

subscript position

$$\vec{I} = (I_1, I_2, \cdots, I_d)$$

$$\vec{L} = (L_1, L_2, \cdots, L_d)$$

$$\vec{U} = (U_1, U_2, \cdots, U_d)$$

$$\vec{L} \le \vec{U}$$

linear functions

$$b_0 + b_1 I_1 + b_2 I_2 + \cdots + b_d I_d$$

Problem Formulation

- Dependence will exist if there exists two iteration vectors $\vec{k}$ and $\vec{j}$ such that $\vec{L} \le \vec{k} \le \vec{j} \le \vec{U}$ and:

$$
\begin{aligned}
& f_1(\vec{k}) = g_1(\vec{j}) \\
\text{and}\quad & f_2(\vec{k}) = g_2(\vec{j}) \\
\text{and}\quad & \vdots \\
\text{and}\quad & f_m(\vec{k}) = g_m(\vec{j})
\end{aligned}
$$

- That is:

$$
\begin{aligned}
& f_1(\vec{k}) - g_1(\vec{j}) = 0 \\
\text{and}\quad & f_2(\vec{k}) - g_2(\vec{j}) = 0 \\
\text{and}\quad & \vdots \\
\text{and}\quad & f_m(\vec{k}) - g_m(\vec{j}) = 0
\end{aligned}
$$

Problem Formulation - Example

```
      do i = 2, 4
S1:     a(i) = b(i) + c(i)
S2:     d(i) = a(i-1)
      end do
```

- Does there exist two iteration vectors i_1 and i_2, such that
  $2 \leq i_1 \leq i_2 \leq 4$ and such that:

  $i_1 = i_2 - 1$?
- Answer: yes; $i_1=2$ & $i_2=3$ and $i_1=3$ & $i_2=4$.
- Hence, there is dependence!
- The dependence distance vector is $i_2-i_1 = 1$.
- The dependence direction vector is $\text{sign}(1) = <$.

Problem Formulation - Example

```
      do i = 2, 4
S1:     a(i) = b(i) + c(i)
S2:     d(i) = a(i+1)
      end do
```

- Does there exist two iteration vectors i_1 and i_2, such that
  $2 \leq i_1 \leq i_2 \leq 4$ and such that:

  $i_1 = i_2 + 1$?
- Answer: yes; $i_1=3$ & $i_2=2$ and $i_1=4$ & $i_2=3$. (But, but!).
- Hence, there is dependence!
- The dependence distance vector is $i_2-i_1 = -1$.
- The dependence direction vector is sign(-1) = >.
- Is this possible?

Problem Formulation - Example

```
      do i = 1, 10
S1:     a(2*i) = b(i) + c(i)
S2:     d(i) = a(2*i+1)
      end do
```

- Does there exist two iteration vectors i_1 and i_2, such that
  $1 \leq i_1 \leq i_2 \leq 10$ and such that:

  $$2*i_1 = 2*i_2 + 1?$$
- Answer: no; $2*i_1$ is even & $2*i_2+1$ is odd.
- Hence, there is no dependence!

Problem Formulation

- Dependence testing is equivalent to an integer linear programming (ILP) problem of 2d variables & m+d constraint!
- An algorithm that determines if there exits two iteration vectors $\vec{k}$ and $\vec{j}$ that satisfies these constraints is called a dependence tester.
- The dependence distance vector is given by $\vec{j} - \vec{k}$
- The dependence direction vector is give by sign($\vec{j} - \vec{k}$).
- Dependence testing is NP-complete!
- A dependence test that reports dependence only when there is dependence is said to be exact. Otherwise it is in-exact.
- A dependence test must be conservative; if the existence of dependence cannot be ascertained, dependence must be assumed.

Dependence Testers

- Lamport’s Test.
- GCD Test.
- Banerjee’s Inequalities.
- Generalized GCD Test.
- Power Test.
- I-Test.
- Omega Test.
- Delta Test.
- Stanford Test.
- etc…

CSC D70:
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Winter 2018

The content of this lecture is adapted from the lectures of Todd Mowry and Tarek Abdelrahman